2015
HSC
ASSESSMENT TASK 4

Trial HSC Examination

Mathematics

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General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using a blue or black pen.
- Board approved calculators and mathematical templates and instruments may be used.
- Show all necessary working in Questions 11 – 16.
- This examination booklet consists of 15 pages including a standard integral page and multiple choice answer sheet.

Total marks (**100**)

Section I

Total marks (**10**)

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided.
- Allow 15 minutes for this section.

Section II

Total marks (**90**)

- Attempt questions 11 – 16
- Answer each question in the Writing Booklets provided.
- Start a new booklet for each question with your name and question number at the top of the page.
- All necessary working should be shown for every question.
- Allow 2 hours 45 minutes for this section.

Name: ____

Teacher: ____

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

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1. The solution to $3x^2 - 7x = 1$ is

(A) $\dfrac{-7 \pm \sqrt{37}}{6}$ (B) $\dfrac{7 \pm \sqrt{61}}{6}$ (C) $\dfrac{-7 \pm \sqrt{61}}{6}$ (D) $\dfrac{7 \pm \sqrt{37}}{6}$

2. The maximum value of the expression $-2x^2 - 4x + 7$ is

(A) -1 (B) 1 (C) 7 (D) 9

3. A primitive function for $12x^2 - 4$ could be

(A) $24x$ (B) $4x^3 + 1$ (C) $24x + 10$ (D) $4x^3 - 4x + 1$

4. A function is defined by the following rule:

$$f(x) = \begin{cases} 0 & \text{if} \quad x \le -2 \\ -1 & \text{if} \quad -2 < x < 0 \\ x & \text{if} \quad x \ge 0 \end{cases}$$

What is the value of $f\left(a^2\right)$?

(A) a^2 (B) x (C) -1 (D) 0

5. The domain of the curve $y = \log_e(x+1)$ is

(A) $x < 1$ (B) $x > -1$ (C) $x \ge -1$ (D) $x > 1$

6. The derivative of $y = \log_4 x$ is

(A) $\frac{1}{x}$ (B) $\frac{1}{x \ln 4}$ (C) $\frac{4}{x}$ (D) $\frac{\ln 4}{x}$

7. In a large business the employees are 55% male and 45% female. Two employees are selected at random. What is the probability that both are male?

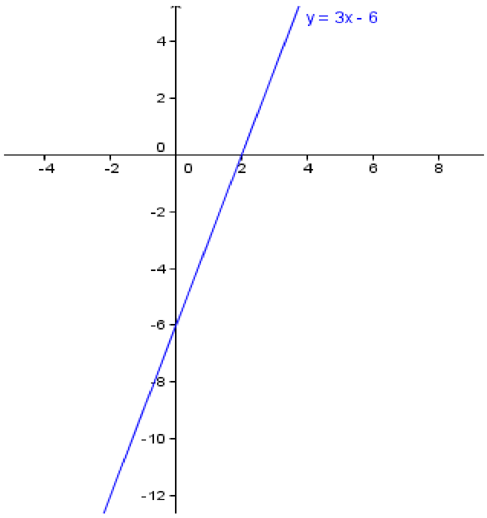
(A) 0.2025 (B) 0.2475 (C) 0.3025 (D) 0.5555

8. What is solution to the equation $\frac{\cos\theta}{\sqrt{3}} = -\frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$?

(A) $\theta = 30^\circ$ or 330° (B) $\theta = 60^\circ$ or 300°

(C) $\theta = 120^\circ$ or 240° (D) $\theta = 150^\circ$ or 210°

9.



The graph of the line $y = 3x - 6$ is shown above. The line $y = 3x - 6$ is moved horizontally 1 unit to the right. The resulting line will have an equation:

(A) $y = 3x + 9$ (B) $y = 3x - 7$

(C) $y = 3x - 9$ (D) $y = 3x - 8$

10. The second term of an arithmetic series is 37 and the sixth term is 17. What is the sum of the first ten terms?

(A) 54 (B) 195 (C) 280 (D) 390

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) Start a new booklet **Marks**

(a) Factorise $x^2 - 4y^2 - x + 2y$. **2**

(b) Graph on a number line, the solution to $|x+2| < 3$. **2**

(c) Simplify $\frac{1}{3\sqrt{2}-1} + \frac{2}{3\sqrt{2}+1}$. **2**

(d) For what values of m will $2x^2 + mx + 4 = 0$ have real and distinct roots? **2**

(e) Consider the parabola with focus $(1, -2)$ and directrix $y = 4$.

(i) Find the co-ordinates of the vertex. **1**

(ii) Hence, find the equation of the parabola. **1**

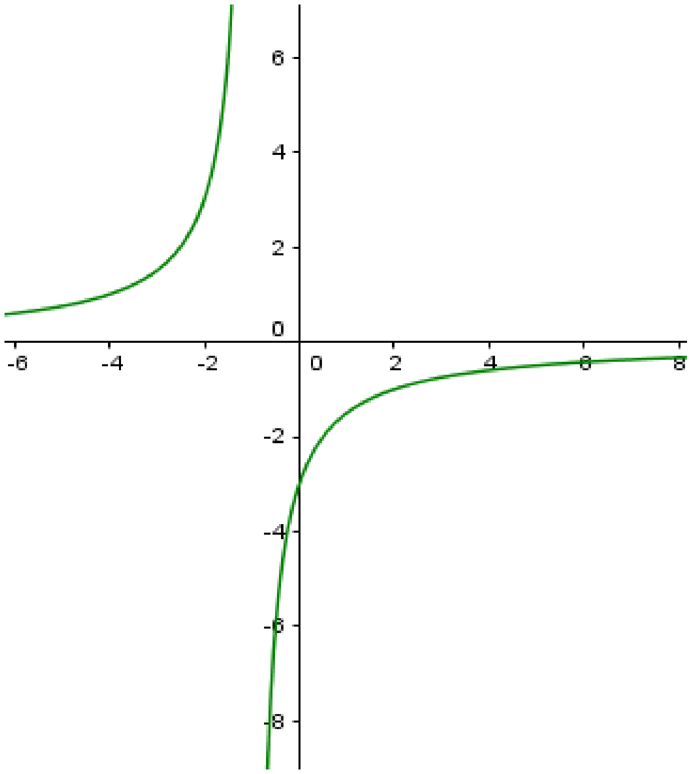
(f) A and B are the points $(0,1)$ and $(0,7)$ respectively. The point $P(x, y)$ moves so that the distance PA is equal to twice the distance PB.
Show that the equation of the locus of P is given by $x^2 + y^2 - 18y + 65 = 0$. **2**

Question 11 continues on the next page ……

Marks

Question 11 continued ……

(g) The diagram below shows the graph of the hyperbola $y = \dfrac{-3}{x+1}$.



(i) Show that $y = \dfrac{-3}{x+1}$ and the line $y = 2$ intersect when $x = -\dfrac{5}{2}$. **1**

(ii) Copy the diagram on to your answer booklet.

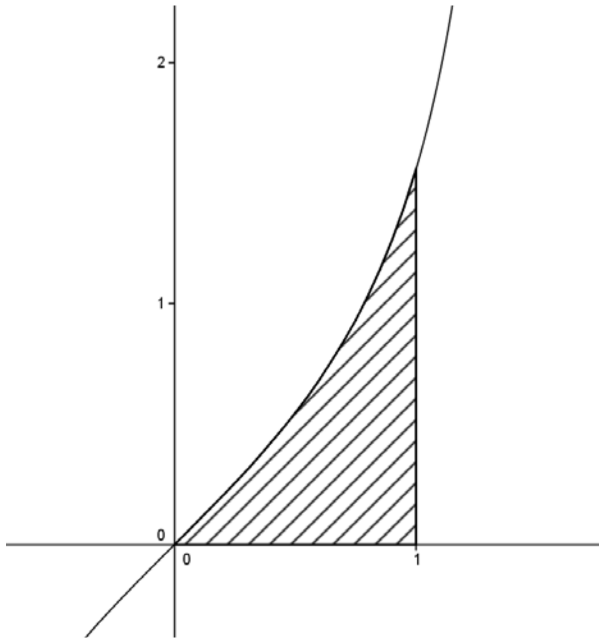
Using the diagram, state for what values of x, $\dfrac{-3}{x+1} > 2$. **2**

Question 12 (15 marks) Start a new booklet **Marks**

(a) Determine if the function $f(x) = x^4 - 16$ is an even function, an odd function or neither. Justify your answer. **2**

(b) Differentiate $y = (x^4 - 1)^9$ and hence find $\int x^3 (x^4 - 1)^8 \, dx$. **2**

(c) (i) The graph below shows the area under the curve $y = \tan x$ (where x is measured in radians) between $x = 0$ and $x = 1$, and above the x axis.



Write three inequalities to describe the region inside the triangle. **3**

(ii) Use Simpson's rule, with three function values, to calculate the volume of the solid generated when the region in part (i) is rotated about the x-axis. Give your answer to one decimal place. **3**

(d) The function $f(x)$ is defined by the rule $f(x) = x^3 - 3x^2$ in the domain $0 \le x \le 4$.

i) It is given, that $y = f(x)$ has a maximum turning point at $(0,0)$ and a minimum turning point at $(2,-4)$. Draw a neat sketch of the graph $y = f(x)$, showing clearly the turning points, the intercepts with the x-axis and the y-axis and the values at the extremities of the domain. **2**

ii) Indicate on your sketch the region bounded entirely by the parts of the graph of $y = f(x)$ and the x-axis. Find the area of this region. **3**

Question 13 (15 marks) Start a new booklet **Marks**

(a) Solve for *x*, giving answer to 2 significant figures :

$$3e^{10x} - 5 = 0$$ **3**

(b) (i) Show that the derivative of $2xe^{-x^2} + 1$ is given by $2e^{-x^2}\left(1-2x^2\right)$. **2**

(ii) Find the equation of the normal to the curve $y = 2xe^{-x^2} + 1$, at the point where it crosses the *y* axis. **2**

(c) Find the volume of the solid formed by rotating the area bounded by $y = \log_e x$, the *x* and *y* axis and the line $y = \log_e 3$ about the *y* axis. **3**

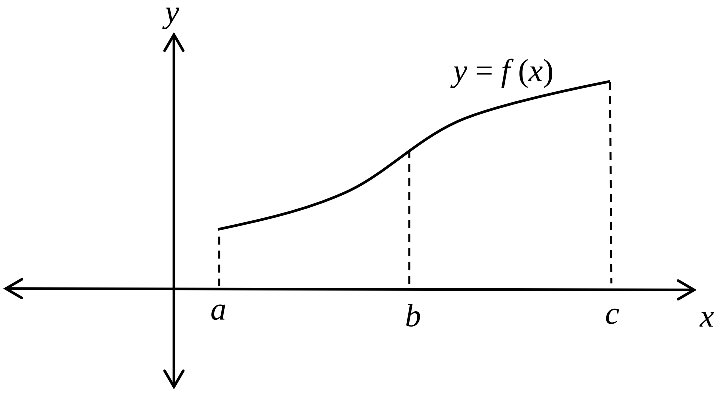
(d) Differentiate $\log_e\left(\dfrac{\sqrt{x}}{2x+1}\right)$ by using the log laws. **3**

(e) Find $\displaystyle\int_1^3 \frac{2x}{x^2+3}\,dx$ **2**

Question 14 (15 marks) Start a new booklet **Marks**

(a) Differentiate: $\dfrac{4x^5 - 2}{1 - x}$ **2**

(b)



The diagram above shows the graph of the function $y = f(x)$ over the domain $a \le x \le c$.

(i) Name the feature at $x = b$. **1**

(ii) Discuss the behavior of $y = f'(x)$ and $y = f''(x)$ over the given domain. **3**

(c) For the function $f(x) = x^4 - 8x^3 + 24x^2 - 32x + 19$:

(i) show that $f'(2) = f''(2) = 0$. **2**

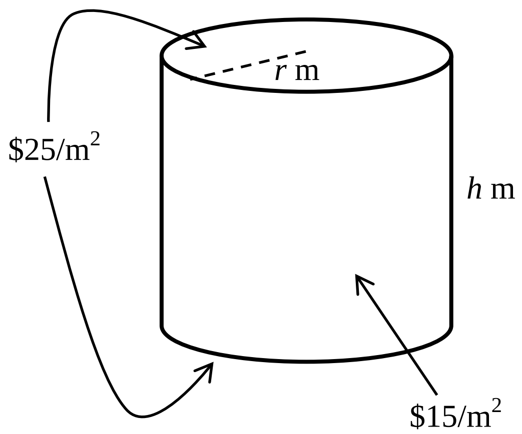
(ii) find the co-ordinates of the stationary point on $y = f(x)$ and determine its nature. **2**

Question 14 continues on the next page ……

Marks

Question 14 continued ……

(d)



A cylindrical plastic water tank of radius r metres, height h metres and capacity 4 000 litres can be manufactured for $\$25/\text{m}^2$ for the top and bottom and $\$15/\text{m}^2$ for the tank wall. These costs include both materials and the manufacturing process.

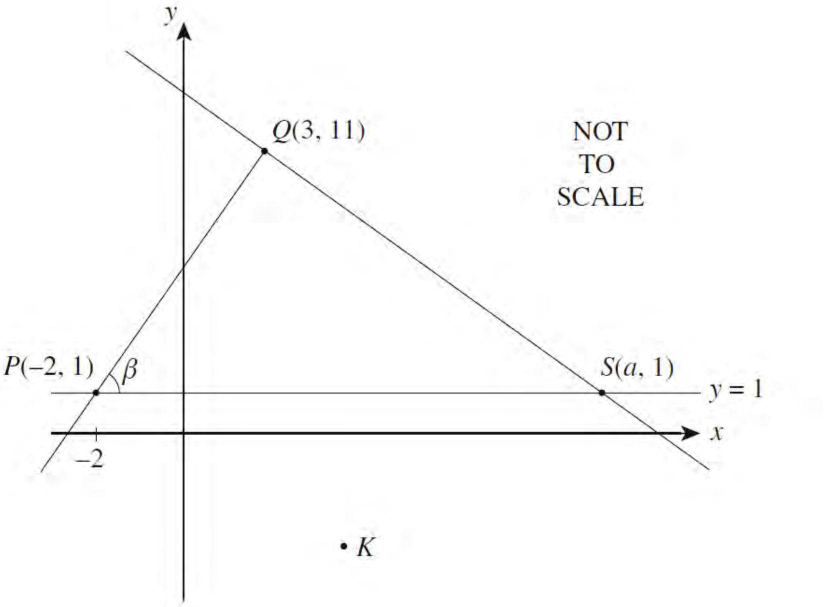
(i) Show the cost, $C, of making this tank is given by the formula: **2**

$$C = \$\left(50\pi r^2 + \frac{120}{r}\right)$$

(ii) Find the dimensions of the tank for cost to be a minimum and hence, find this cost to the nearest dollar. **3**

Question 15 (15 marks) Start a new booklet **Marks**

(a)



In the diagram P is the point $(-2,1)$ and Q is the point $(3,11)$. Both points P and S lie on the line $y=1$. Point S has coordinates $(a,1)$. K is a point in the fourth quadrant. Line PQ makes an angle of β with the line $y=1$.

(i) Find the gradient of *PQ*. **1**

(ii) Find the equation of the line *PQ*. **1**

(iii) Briefly explain why $\beta = 63^0$ correct to the nearest degree. **1**

(iv) *PQSK* is a rhombus. Find the **exact** lengths of *PQ* and *PK*. Give a geometrical reason for your answer. **2**

(v) Calculate the size of $\angle PQS$. Give a reason for your answer. **2**

(vi) By using the properties of a rhombus, or otherwise, show that the value of *a*, the *x*-coordinate of point *S*, is 8. **2**

Question 15 continues on the next page ……

Marks

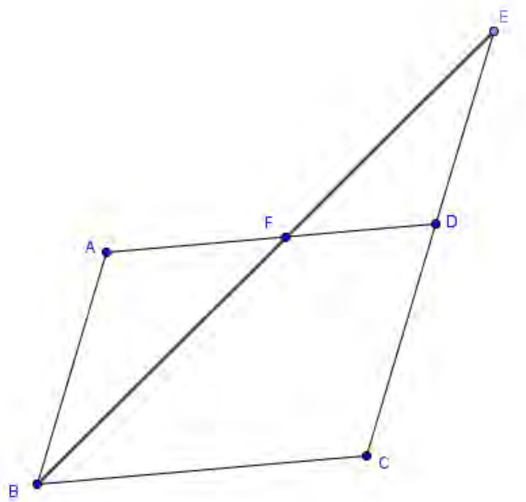
Question 15 continued ……

(b) A person invests \$600 at the beginning of each year in a superannuation fund. Compound interest is paid at 8% per annum on the investment. The first \$600 is to be invested at the beginning of 2008 and the last is to be invested at the beginning of 2037. Calculate to the nearest dollar:

(i) The amount to which the 2008 investment will have grown by the beginning of 2038. **1**

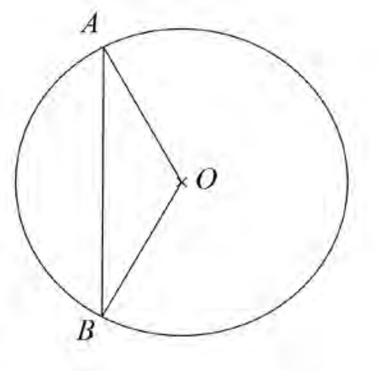
(ii) The amount to which the total investment will have grown by the beginning of 2038. **3**

(c) In the diagram below, *ABCD* is a parallelogram with *CD* produced to *E*. *BE* meets *AD* at *F*. Prove that $\Delta ABF \;|||\; \Delta DEF$. **2**



Question 16 (15 marks) Start a new booklet **Marks**

(a)



A circle has centre O and radius of 12 cm. The length of arc AB is 8π cm.

(i) What is the size of $\angle AOB$? Answer in radians. **1**

(ii) Find the area of the minor segment cut off by the chord AB.
Give your answer to one decimal place. **2**

(b) Alex and Bella leave from point O at the same time.
Alex travels at 20 km/h along a straight road in the direction 085° T.
Bella travels at 25 km/h along another straight road in the direction 340° T.

Draw a diagram to represent this information.

(i) Show that $\angle AOB$ is 105° where $\angle AOB$ is the angle between the directions taken by Alex and Bella. **1**

(ii) Find the distance Alex and Bella are apart, to the nearest kilometre, after two hours. **2**

(c) Prove $(\sec\theta - \cos\theta)^2 = \tan^2\theta - \sin^2\theta$ **2**

Question 16 continues on the next page ……

Marks

Question 16 continued ……

(d) Four cards, numbered 3, 4, 5 and 6 are used in a game.

3	**4**	**5**	**6**

The four cards are placed face down and each player pays \$1 to take a turn to draw two cards, one at a time without replacement.

The two cards make a two digit number, with the first card drawn being the first digit of their number.

If the cards form a number over 60, the player receives \$3 back (i.e. they win \$2), otherwise, they receive nothing back.

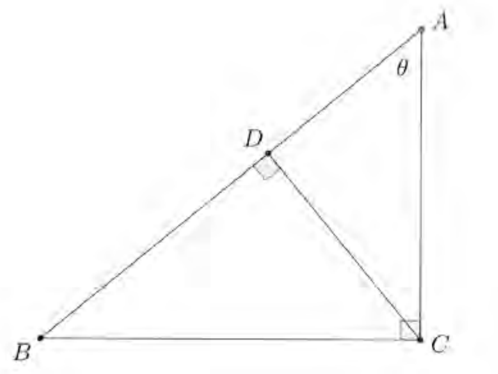
The four cards are then shuffled and replaced for the next turn.

(i) Use a diagram to show all of the possible 2-digit numbers that could be drawn. **1**

(ii) What is the probability that a player will win on their first turn? **2**

(iii) Calculate the probability that a player who brings \$5 to play will have \$3 left after 5 turns? **2**

(e) A triangle *ABC* is right-angled at *C*.
D is the point on *AB* such that *CD* is perpendicular to *AB*.
Let $\angle BAC = \theta$.



Given that $8AD + 2BC = 7AB$.

Show that $8\cos\theta + 2\tan\theta = 7\sec\theta$ **2**

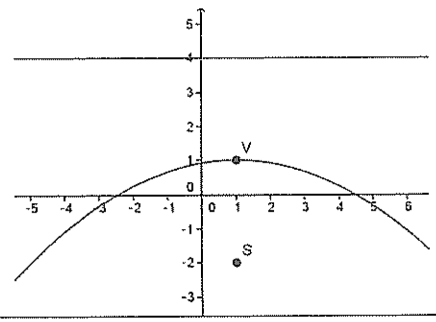
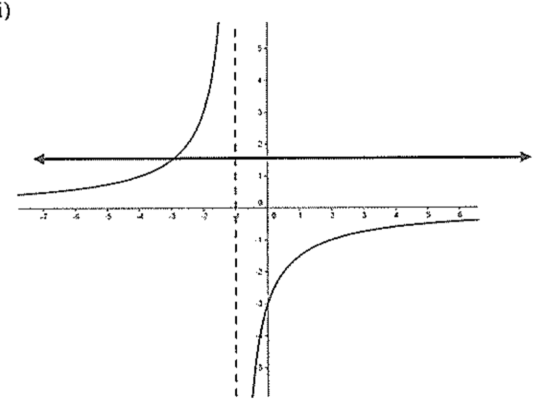
End of Examination

Year 12 Trial Question No.11	Mathematics Solutions and Marking Guidelines	Examination 2015

Outcomes Addressed in this Question

P3 Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.

P4 Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.

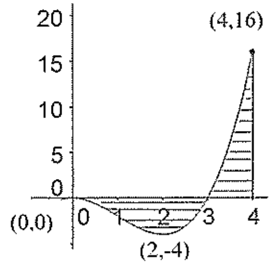
Outcome	Solutions	Marking Guidelines
P4	(a) x^2-4y^2-x+2y $=(x-2y)(x+2y)-1(x-2y)$ $=(x-2y)(x+2y-1)$	2 marks : correct answer 1 mark : substantial progress towards correct solution
P4	(b) $\lvert x+2\rvert<3$ $\therefore -3<x+2<3$ $\therefore -5<x<1$ 	2 marks : correct solution 1 mark : substantial progress towards correct solution
P3	(c) $\frac{1}{3\sqrt{2}-1}+\frac{2}{3\sqrt{2}+1}$ $=\frac{3\sqrt{2}+1\ +\ 2(3\sqrt{2}-1)}{(3\sqrt{2}-1)(3\sqrt{2}+1)}$ $=\frac{9\sqrt{2}-1}{17}$	2 marks : correct solution 1 mark : substantial progress towards correct solution
P4	(d) Given $2x^2+mx+4=0$, $\Delta=m^2-4\times2\times4$. $\therefore\Delta=m^2-32$ Real and distinct roots when $\Delta>0$. Solving $m^2-32>0$, $(m-\sqrt{32})(m+\sqrt{32})>0$. Graphing this concave up parabola gives $m<-\sqrt{32}$ and $m>\sqrt{32}$.	2 marks : correct solution 1 mark : substantial progress towards correct solution
P4	(e) 	
	(i) Vertex is half way between focus and directrix. From the graph, V is $(1,1)$. (ii) As concave down parabola, equation is in the form $(x-1)^2=-4a(y-1)$ and $a=3$ (focal length) $\therefore$ parabola is $(x-1)^2=-12(y-1)$	1 mark : correct answer 1 mark : correct answer
P4	(f) distance PA is equal to twice the distance PB $\therefore\sqrt{(x-0)^2+(y-1)^2}=2\left(\sqrt{(x-0)^2+(y-7)^2}\right)$ $\therefore x^2+(y-1)^2=4\left(x^2+(y-7)^2\right)$ $\therefore x^2+y^2-2y+1=4x^2+4y^2-56y+196$ $\therefore 0=3x^2+3y^2-54y+195$ $\therefore x^2+y^2-18y+65=0$	2 marks : correct solution 1 mark : substantial progress towards correct solution
P3	(g) (i) $y=\frac{-3}{x+1}$ and $y=2$ meet when $2=\frac{-3}{x+1}$ $\therefore 2x+2=-3$ $\therefore 2x=-5$ $\therefore x=-\frac{5}{2}$.	1 mark: correct solution
P4	(ii)  From part (i), the graphs of $y=\frac{-3}{x+1}$ and $y=2$ intersect when $x=-\frac{5}{2}$. $\therefore$ the graph of $y=\frac{-3}{x+1}$ is above the graph of $y=2$ when $\frac{-5}{2}<x<-1$.	2 marks : correct solution 1 mark : substantial progress towards correct solution or indicates the line or equivalent on the diagram, so that the solution can be read from the diagram.

Year 12 2015	Mathematics	Task 4 Yearly
Question No. 12	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

P5 understands the concept of a function and the relationship between a function and its graph

H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
	Question 12	
P5	a) $f(x)=x^4-16$ $f(-x)=(-x)^4-16$ $=x^4-16$ Since $f(x)=f(-x)=x^4-16$, then the function $f(x)=x^4-16$ is an even function.	2 Marks for complete correct solution with complete correct reasoning and using correct terminology. 1 Mark for partial correct solution
H8	b) $\frac{dy}{dx}=9(4x^3)(x^4-1)^8$ $=36x^3(x^4-1)^8$ $\therefore \int x^3(x^4-1)^8\,dx$ $=\frac{1}{36}\int 36x^3(x^4-1)^8\,dx$ $=\frac{1}{36}(x^4-1)^9+C$	2 Marks for complete correct solution 1 Mark for partial correct solution
H8	c) (i) $y \leq \tan x, \quad x \leq 1, \quad y \geq 0$	1 mark for $y \geq \tan x$ or $y > \tan x$ 1 mark for $x \leq 1$ or $x < 1$ 1 mark for $y \leq 0$ or $y < 0$
H8	(ii) $V \approx \pi\int_0^1 \tan^2 x\ dx$ $\approx \pi\left(\frac{\left(\frac{1-0}{2}\right)}{3}\right)\left(\tan^2(0)+4\tan^2\left(\frac{1}{2}\right)+\tan^2(1)\right)$ $\approx 1\cdot 895...$ $\approx 1\cdot 9$ (to one decimal place)	3 Marks for complete correct solution 2 Marks for correct solution but forgetting π 1 mark for any correct working that could lead to a solution.
H8	(d) (i)  	2 Marks for complete correct graph labelling clearly the turning point, intercepts and extremities. 1 mark for correct graph without labelling clearly the turning point, intercepts and extremities.
H8	(ii) Area $=\left\lvert\int_0^3 (x^3-3x^2)\,dx\right\rvert+\int_3^4 (x^3-3x^2)\,dx$ $=\left\lvert\left[\frac{x^4}{4}-\frac{3x^3}{3}\right]_0^3\right\rvert+\left[\frac{x^4}{4}-\frac{3x^3}{3}\right]_3^4$ $=\left\lvert\left[\frac{x^4}{4}-x^3\right]_0^3\right\rvert+\left[\frac{x^4}{4}-x^3\right]_3^4$ $=\left\lvert\left(\frac{(3)^4}{4}-3^3\right)-\left(\frac{(0)^4}{4}-0^3\right)\right\rvert+\left(\left(\frac{(4)^4}{4}-4^3\right)-\left(\frac{(3)^4}{4}-3^3\right)\right)$ $=\frac{27}{4}+\frac{27}{4}$ $=\frac{27}{2}$ units^2	3 Marks for complete correct solution 2 Marks for a substantially correct solution with only one error 1 Mark for integrating correctly

Multiple Choice Answers:

1. B 2. D 3. D 4. A 5. B 6. B 7. C 8. D 9. C 10. B

Year 12 Mathematics Trial 2015

Question No. 13 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

Manipulates algebraic expressions involving logarithmic and exponential functions

Outcome	Solutions	Marking Guidelines

(a)

$3e^{10x} - 5 = 0$

$e^{10x} = \frac{5}{3}$

$\ln\left(e^{10x}\right) = \ln\left(\frac{5}{3}\right)$

$10x = \ln\left(\frac{5}{3}\right)$

$x = \frac{\ln\left(\frac{5}{3}\right)}{10}$

$\therefore x = 0.051$ (2 sig figs)

3 marks Correct solution with correct rounding.
2 marks Substantial progress towards correct solution
1 mark Some progress towards correct solution

(b) (i)

$y = 2xe^{-x^2} + 1$

Let $u = 2x$ and $v = e^{-x^2}$

Then $u' = 2$ and $v' = -2xe^{-x^2}$

$y' = uv' + vu'$

$y' = 2x.(-2xe^{-x^2}) + e^{-x^2}.2$

$y' = 2e^{-x^2} - 4x^2e^{-x^2}$

$\therefore y' = 2e^{-x^2}(1-2x)$

2 marks Correct solution.
1 mark Substantial progress towards correct solution.

(ii)

$y = 2xe^{-x^2} + 1$

Crosses the y-axis when $x = 0$. i.e at $(0,1)$

When $x = 0$, $y' = 2e^0(1-0)$

$y' = 2$ that is, gradient of tangent is 2.

$\therefore$ gradient of normal is $\frac{-1}{2}$.

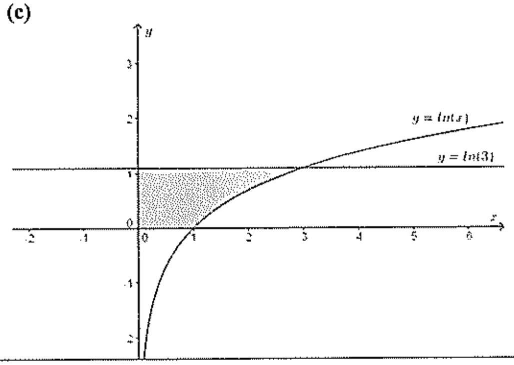
Equation of normal:

$y - 1 = \frac{-1}{2}(x - 0)$

$\therefore y = \frac{-1}{2}x + 1$ or $x + 2y - 2 = 0$

2 marks Correct solution.
1 mark Substantial progress towards correct solution.

(c)



$y = \log_e x$

$x = e^y$

$\therefore x^2 = e^{2y}$

$V = \pi \int_0^{\log_e 3} e^{2y}\, dy$

$= \pi \left[\frac{e^{2y}}{2}\right]_0^{\log_e 3}$

$= \frac{\pi}{2}\left(e^{2\log_e 3} - e^0\right)$

$= \frac{\pi}{2}\left(e^{\log_e 3^2} - 1\right)$

$= \frac{\pi}{2}(9-1)$

$= 4\pi$ units^3

3 marks Correct solution
2 marks Substantial progress towards correct solution
1 mark Some progress towards correct solution

(d)

$\log_e\left(\frac{\sqrt{x}}{2x+1}\right) = \log_e \sqrt{x} - \log_e(2x+1)$ by log laws

$= \log_e x^{\frac{1}{2}} - \log_e(2x+1)$

$= \frac{1}{2}\log_e x - \log_e(2x+1)$

$\therefore \frac{d}{dx} = \frac{1}{2x} - \frac{2}{2x+1}$

3 marks Correct solution
2 marks Substantial progress towards correct solution
1 mark Some progress towards correct solution

(e)

$\int_1^3 \frac{2x}{x^2+3}\, dx = \left[\log_e(x^2+3)\right]_1^3$

$= \left[\log_e(9+3) - \log_e(1+3)\right]$

$= \log_e 12 - \log_e 4$

$= \log_e \frac{12}{4}$

$= \log_e 3$

2 marks Correct solution.
1 mark Substantial progress towards correct solution.

Year 12 Mathematics Trial Examination 2015

Question No. 14 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

H4 expresses practical problems in mathematical terms based on simple given models
H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
H7 uses the features of a graph to deduce information about the derivative

Outcome	Solutions	Marking Guidelines
H5	(a) $\frac{d}{dx}\frac{4x^5-2}{1-x} = \frac{(1-x).20x^4-(4x^5-2).-1}{(1-x)^2}$ $= \frac{20x^4-20x^5+4x^5-2}{(1-x)^2}$ $= \frac{20x^4-16x^5-2}{(1-x)^2}$	**2 marks** Correct solution. **1 mark** Uses quotient rule correctly.
H7	(b) (i) Point of inflexion	**1 mark** Correct answer.
H5, H7	(ii) Function increasing $\therefore f'(x)>0,\quad a\le x\le c$ Concave up $\therefore f''(x)>0,\quad a\le x\le b$ Point of inflexion $\therefore f''(x)=0,\quad x=b$ Concave down $\therefore f''(x)<0,\quad b\le x\le c$	**3 marks** Signs of derivatives correct, domains correct. **2 marks** Incomplete discussion, one sign/domain incorrect. **1 mark** At least 1 sign and domain correct.
H5	(c) (i) $f(x) = x^4-8x^3+24x^2-32x+19$ $f'(x) = 4x^3-24x^2+48x-32$ $f''(x) = 12x^2-48x+48$ $f'(2) = 4\times 8-24\times 4+48\times 2-32$ $=0$ $f''(2) = 12\times 4-48\times 2+48$ $=0$	**2 marks** Correctly evaluates both derivatives. **1 mark** Only one of the derivatives evaluated correctly OR correctly finds the first and second derivative without evaluating.
H4	(ii) Since $f'(2)=0 \Rightarrow$ the stationary point is at $x=2$. $f(2)=3$ ie. Stationary point at (2,3) Since $f''(2)=0 \Rightarrow$ possible point of inflexion is at $x=2$ Check for sign change x: 1, 2, 3 $f''(x)$: 12, 0, 12 No sign change in $f''(x)$, therefore, no point of inflexion. Since $f''(x)$ is positive either side of stationary point, the curve must be concave up and hence a minimum turning point at (2,3).	**2 marks** Correct solution. **1 mark** correct co-ordinates only or correct nature with justification.
H4, H5	(d) (i) $V = 4\text{m}^3$ ie. $\pi r^2 h = 4$ $h = \frac{4}{\pi r^2}$ Cost of top and bottom of tank $C_1 = 2\pi r^2 \times \$25$ $= 50\pi r^2$ Cost of wall of tank $C_2 = 2\pi rh \times 15$ $= 30\pi rh$ $= 30\pi r \times \frac{4}{\pi r^2}$ $= \frac{120}{r}$ Total cost $\quad C = 50\pi r^2 + \frac{120}{r}$	**2 marks** Correct solution. **1 mark** correctly finds cost function for top and bottom or wall.
H4, H5	(ii) $\frac{dC}{dr} = 100\pi r - \frac{120}{r^2}$ $\frac{d^2C}{dr^2} = 100\pi + \frac{240}{r^3}$ since $r>0$, any stationary points will be MINIMUM. C will be stationary when $\frac{dC}{dr}=0$ ie $100\pi r - \frac{120}{r^2} = 0$ $100\pi r = \frac{120}{r^2}$ $r^3 = \frac{120}{100\pi}$ $r = 0.725\text{m}$ also, $h = \frac{4}{\pi(0.725)^2}$ $\approx 2.42\text{m}$ (correct to 2 dec. pl.) $C = 50\pi(0.725)^2 + \frac{120}{(0.725)}$ $= \$248$ (to the nearest dollar.)	**3 marks** Correct and complete solution with full justification where required. **2 marks** Substantial progress towards correct solution. **1 mark** Some progress towards a correct solution.

Year 12 Assessment Task4 Question No.15	Mathematics Solutions and Marking Guidelines	Examination 2015

Outcomes Addressed in this Question

H2 - constructs arguments to prove and justify results

H5 - applies appropriate techniques from the study of geometry and ,series to solve problems

Solutions	Marking Guidelines
(a)	
(i) Gradient of PQ $m_{PQ} = \frac{11-1}{3-(-2)} = 2$	**Award 1** for correct solution
(ii) Equation of line PQ $m_{PQ} = 2$ point $(-2,1)$ $y-1 = 2(x-(-2))$ $y = 2x+5$	**Award 1** for correct solution
(iii) $\beta = 63^o$ because $m = 2$ and $\tan 63^o = 2$	**Award 1** for correct solution
(iv) $PQSK$ is a rhombus and all sides of a rhombus are equal PQ and PK are sides of a rhombus. $\therefore PQ = PK$ $PQ = \sqrt{(11-1)^2 + (3-(-2))^2}$ $= \sqrt{100+25} = \sqrt{125}$ $\therefore PK = \sqrt{125}$	**Award 2** for correct solution **Award 1** for substantial progress towards correct solution
(v) In ΔPQS, $PQ = QS$ ($PQSK$ is a rhombus and all sides of a rhombus are equal) $\therefore \Delta PQS$ is Isosceles $\angle QPS = \angle QSP$ (angles opposite equal sides are equal in an Isosceles triangle) $\angle QPS = 63^o$ (proven in (iii)) $\therefore \angle QSP = 63^o$ $\angle QPS + \angle QSP + \angle PQS = 180^o$ (angle sum of a triangle equals 180^o) $63^o + 63^o + \angle PQS = 180^o$ $\therefore \angle PQS = 54^o$	**Award 2** for correct solution **Award 1** for substantial progress towards correct solution
(vi) The diagonals of a rhombus intersect at 90^o $\therefore$ They will intersect at (3,1). The length from (-2,1) to (3,1) is 5 units. $\therefore$ From (3,1) to S has to be 5 units (Diagonals of a rhombus bisect each other) $\therefore a = 3+5$ $a = 8$	**Award 2** for correct solution **Award 1** for substantial progress towards correct solution
(b)	
(i) $P = \$600$, $r = 0.08$, $n = 30$ $A = P(1+r)^n$ $= 600 \times (1+0.08)^{30}$ $= 6037.60$	**Award 1** for correct solution
(ii) The initial \$600 invested at the beginning of 2008 will grow to $A_1 = 600 \times (1.08)^{30}$ by 2038 The second \$600 invested at the beginning of 2009 will grow to $A_2 = 600 \times (1.08)^{29}$ by 2038 The third \$600 invested at the beginning of 2010 will grow to $A_3 = 600 \times (1.08)^{28}$ by 2038 The final \$600 invested at the beginning of 2037 will grow to $A_{30} = 600 \times (1.08)^1$ by 2038 Total investement $= A_1 + A_2 + A_3 + \dots\dots + A_n$ $= 600 \times (1.08^{30} + 1.08^{29} + 1.08^{28} + \dots + 1.08^1)$ $= 600 \times \frac{1.08(1.08^{30}-1)}{1.08-1} = \$73407.52$	**Award 3** for correct solution **Award 2** for substantial progress towards correct solution **Award 1** for progress towards solution
(c) In ΔABF and ΔDEF $\angle AFB = \angle EFD$ (Vertically opposite angles are equal) $AB \parallel CE$ (opposite sides of parallelogram $ABCD$ are parallel , CD is produced to CE) $\angle FAB = \angle FDE$ (Alternate angles on parallel lines are equal, $AB \parallel CE$) $\therefore \Delta ABF \;\vert\vert\vert\; \Delta DEF$ (equiangular)	**Award 2** for correct solution **Award 1** for substantial progress towards correct solution

Year 12 Question 16	Mathematics Solutions and Marking Guidelines	Task 4 2015

Outcome Addressed in this Question

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

Part	Solutions	Marking Guidelines

(a) (i)

$l = r\theta$

$8\pi = 12\theta$

$\theta = \dfrac{2\pi}{3}$

1 Mark: Correct answer.

(ii)

$A = \frac{1}{2} \times r^2 \times \theta$

$= \frac{1}{2} \times 12^2 \times \frac{2\pi}{3}$

$= 48\pi \text{ cm}^2$

$A = \frac{1}{2} ab \sin O$

$= \frac{1}{2} \times 12 \times 12 \times \sin \frac{2\pi}{3}$

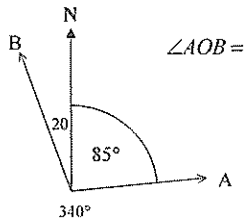
$= 72 \times \frac{\sqrt{3}}{2} = 36\sqrt{3} \text{ cm}^2$

Area of segment $= 48\pi - 36\sqrt{3} \approx 88.44 \text{ cm}^2$

2 Marks: Correct answer.

1 Mark: Determines the area of the sector or triangle.

(b) (i)



$\angle AOB = 20^0 + 85^0 = 105^0$

1 Mark: Correct answer.

(ii)

After 2 hours Alex travels 40 km and Bella travels 50 km.

$AB^2 = 40^2 + 50^2 - 2 \times 40 \times 50 \times \cos 105°$

$AB^2 = 5135.27618...$

$AB = 71.66084133...$

$AB \approx 72$ km

Alex and Bella are 72 km apart after 2 hours.

2 Marks: Correct answer.

1 Mark: Uses the cosine rule with some correct values

(c)

$LHS = (\sec\theta - \cos\theta)^2$

$= \sec^2\theta - 2\sec\theta\cos\theta + \cos^2\theta$

$= 1 + \tan^2\theta - 2 \times 1 + \cos^2\theta$

$= \tan^2\theta - 1 + \cos^2\theta$

$= \tan^2\theta - (1 - \cos^2\theta)$

$= \tan^2\theta - \sin^2\theta$

$= RHS$

2 Marks: Correct solution.

1 Mark: Uses an appropriate identity in an attempt to provide a proof

(d) (i)

	3	4	5	6
3		34	35	36
4	43		45	46
5	53	54		56
6	63	64	65	

1 Mark: All 12 combinations shown.
May use other representations for example, tree diagram.

(ii)

P(win) = P(63 or 64 or 65) = $\frac{3}{12} = \frac{1}{4}$

2 Marks: Correct answer.

1 Mark: Determines the winning possibilities

(iii)

To end up with \$3 the player must win once and lose the other 4 times. There are five combinations of 1 win, 4 losses.

LLLLW
LLLWL
LLWLL
LWLLL
WLLLL

Each of these has the same probability.

So P(\$3 left after 5 games) = $5 \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

$= \frac{405}{1024}$

2 marks: Correct answer.
Decimal or percentage form acceptable

1 Mark: For stating conditions under which player is left with \$3.

(e)

$\cos\theta = \dfrac{AD}{AC}$

$AD = AC\cos\theta$

$\cos\theta = \dfrac{AC}{AB}$

$AB = AC\sec\theta$

$\tan\theta = \dfrac{BC}{AC}$

$BC = AC\tan\theta$

Now $8AD + 2BC = 7AB$

$8AC\cos\theta + 2AC\tan\theta = 7AC\sec\theta$

$8\cos\theta + 2\tan\theta = 7\sec\theta$

2 Marks: Correct answer.

1 Mark: By referring to the diagram, or otherwise, makes some progress towards the solution.